Carrier Frequency Offset Estimation Technique for OFDM Systems

Dr Hussein A. Leftah
School of Electrical Engineering
Basra Engineering Technical College (BETC)

Abstract-
Orthogonal frequency division multiplexing (OFDM) is sensitive to the carrier frequency offset (CFO) due to frequency mismatch between transmitter and receiver local oscillators. Consequently, a new data aided (DA) frequency synchronization algorithm is presented in this paper. The proposed algorithm is able to cope with frequency offsets in the order of multiples of the subcarrier spacing and it could be utilized in time and frequency domains. The simulation results over additive white Gaussian noise (AWGN) channel model confirmed the efficiency of the proposed algorithm in various scenarios of OFDM systems and CFO.

I. INTRODUCTION

Orthogonal frequency division multiplexing (OFDM) that was first proposed by [1] has recently received a significant attractive for its bandwidth efficiency and resilience to multipath channels [2]-[4]. To accomplish the orthogonality on a symbol interval, each subcarrier spaced in frequency exactly the reciprocal of the symbol interval. The OFDM systems are utilizing the discrete Fourier transform (DFT) or its fast algorithm (FFT) counterpart to provide such orthogonality. However, one of the major drawbacks of the OFDM system is its sensitivity to carrier frequency offset (CFO) which models the frequency mismatch between the transmitter and receiver oscillators [5]-[7]. The CFO causes intercarrier interference (ICI), hence destroying the orthogonality of the OFDM subcarriers. Consequently, several methods have been proposed in the last few years in an attempt to minimize the effect of CFO within the system [8]-[11]. These schemes can be categorised into blind and pilot-aided (PA) schemes [12] and [13]. However, PA schemes are found to achieve more reliable and precise CFO estimation than blind schemes. Moose [14] presented a maximum likelihood estimation of the CFO estimation depends on autocorrelation between two identical consecutive OFDM symbols. However, the main drawbacks of this methods is its CFO estimation range between [-0.5 to 0.5] of the subcarrier spacing and loss in the transmission bandwidth as two identical symbols be transmitted. The range of estimation can be widened by increasing number of identical symbols at the cost of estimation accuracy and bandwidth efficiency. Schmidl and Cox [15] present a method of CFO estimation by sending a pseudonoise sequence includes two training symbols. Its main drawbacks is its CFO estimation range between [-0.5 to 0.5] of the subcarrier spacing and the transmitted symbols carry no information data. Algorithm was presented by Morelli and Mengali [16] which increase the CFO estimation range by increasing the number of identical pseudonoise training sequence to L with no loss in accuracy where the range of estimation is • }L/2. This L identical parts can be generated by transmitting a pseudo-noise sequence on the frequencies multiple of L/T and setting zeros on the rest through the inverse FFT. In spite of its successful in DFTOFDM system, its major drawbacks is all the symbols used to transmit a pseudonoise sequence and it is indirect applicable in non-DFT-OFDM system such as discrete cosine transform (DCT) base OFDM. That due to the output of inverse DCT is not a multiple identical sequences when the input on the frequencies multiple of L/T.

In this paper, a method of CFO estimation by the aid of training signals that inserted between information samples is presented. The range of estimation is flexible and depends on training sequence set preamble. Unlike [16]. The transmitted OFDM symbol includes training samples that use in estimation as well as information samples. The CFO is accurately estimated iteratively at the receiver side by a method independently on the FFT transform. Consequently it can be applied in non-DFT-OFDM systems. This algorithm is efficiently working whether the frequency offset is fractional, integer (IFO) or both and it could be applied in both frequency and time domain with slightly modification as shown later. This is so interesting and significant phenomena. The rest of this paper is organized as follows: Section II describes the system model mathematically. The estimation algorithm's mathematical dimensions are laid out in section III. Spectral efficiency analysis is presented in section IV. Simulation results and discussions are presented in section V. Finally, the conclusions are drawn out in section VI.

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II. SYSTEM MODEL

Consider a block-by-block transmission where the complex data symbols are divided into blocks of size N. Each sequence is used to modulate N orthogonal subcarriers during the lth OFDM block $\boldsymbol{X}^l = \left[X_0^l, X_1^l, \ldots\ldots, X_{N-1}^l\right]$ using the inverse discrete Fourier transform (IDFT) or its fast algorithm (IFFT) counterpart. The data symbols are drawn uniformly from the specific constellation such as M-array quadrature amplitude modulation (M-QAM). The result sequence $\boldsymbol{x}^l = \left[x_0^l, x_1^l, \ldots\ldots, x_{N-1}^l\right]$ is then given as

$$\mathbf{x}^T = \mathbf{F}^T\mathbf{x}^l \qquad (1)$$

where **F** is the normalized FFT matrix of size N and T is the transpose operation. Consequently the kth sample of of the sequence can be written as. Hereafter, the block index l will be omitted for the sake of brevity.

The output samples sequence is then formed in groups, each of pilot symbol and data symbols allocated to physically adjacent subcarriers as shown in Fig. 1. Denote P as the total number of pilots in each OFDM symbol and ϱ as a pilots subcarrier position index. Therefore the resultant sequence u that includes both the pilots and the data samples will be of length $M = (P + N)$. For non real (two dimensional 2-D) constellation, the vector u will have both real and imaginary parts $\mathbf{u} = \mathbf{u}^I + j\mathbf{u}^Q$. $N2$ in is equal to $M1$ in this case.

Nyquist sampling period $T_s = \frac{1}{M1\Delta f}$ and the vector in is then serially passed through the LPF as follows

$$u(t)^I = \sum_{m=0}^{M1-1} u(m)^I f(t - \frac{mT}{M1}) \quad (2)$$

$$u(t)^Q = \sum_{m=0}^{M1-1} u(m)^Q f(t - \frac{mT}{M1}) \quad (3)$$

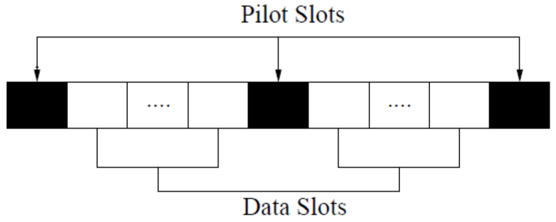

Fig. 1. Signal structure

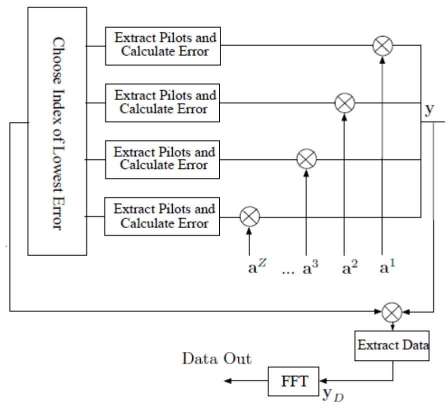

Fig. 2. System block diagram in the presence of CFO cancellation algorithm

III. PROPOSED ALGORITHM

A. *Time domain scenario*

The time domain synchronization system is shown in Fig. 2. The algorithm is based on a known pilot samples inserted between the data samples at the transmitter and on the carrier frequency offset estimation and compensation matrix $\boldsymbol{\Gamma}$ whose zth z = 0, 1, 2, ...,Z − 1 row and nth n = 0, 1, 2, ...,M – 1 element is given as

$$\Gamma(z,n) = e^{j\frac{2\pi v(z)n}{M}}$$

In (2) and (3), the sampling time is chosen to be matched with the FFT. In the case of DCT-II the time shifting is added to filters sampling rate ($T_s = \frac{(2m+1)}{2M1\Delta f}$) to make it mach with DCT-II. The baseband signal is then up-converted to frequency f_c by using a local oscillator and the transmitted signal will be written as

$$r(t) = R[(u(t)^I + ju(t)^Q)e^{j2\pi f_c t}] \quad (4)$$

$$y(t) = r(t) + w \quad (5)$$

Where w denotes the additive white Gaussian noise AWGN. Then the signal y is down converted by local oscillator at the receiver, the in-phase and quadrature phase components of received signal is expressed as

$$y(t)_r^I = [(u(t)^I \cos(2\pi f_c t) - u(t)^Q \sin(2\pi f_c t)) + w(t)] 2\cos(2\pi(f_c - f_0)t) \quad (6)$$

$$y(t)_r^Q = [(u(t)^I \cos(2\pi f_c t) - u(t)^Q \sin(2\pi f_c t)) + w(t)](-2\sin(2\pi(f_c - f_0)t)) \quad (7)$$

After the LPF and sampler the signal will be as

$$y(m)_r^I = \sum_{d=0}^{M-1}[(u(d)^I \cos(2\pi\varepsilon \frac{d}{M1}) - u(d)^Q \sin(2\pi\varepsilon \frac{d}{M1})] + w(m)^I \quad (8)$$

$$y(m)_r^Q = \sum_{d=0}^{M-1}[(u(d)^I \sin(2\pi\varepsilon \frac{d}{M1}) + u(d)^Q \cos(2\pi\varepsilon \frac{d}{M1})] + w(m)^q \quad (9)$$

In (8) and (9), ε is the normalized frequency offset let $\boldsymbol{\Lambda} = diag(e^{j2\pi\varepsilon\frac{m}{M1}})$, $m = (0, 1, 2, \ldots\ldots, M1 - 1)$, then the in-phase and quadrature-phase of received signal could be expressed as

$$\mathbf{y}_r^{I\,T} = \{R[\boldsymbol{\Lambda}\mathbf{u}^T]\} \quad (10)$$

$$\mathbf{y}_r^{Q\,T} = \{Imag[\boldsymbol{\Lambda}\mathbf{u}^T]\} \quad (11)$$

Where $\mathbf{u} = \mathbf{u}^I + j\mathbf{u}^Q$ and $\mathbf{y}_r = \mathbf{y}_r^I + j\mathbf{y}_r^Q + \mathbf{w}$, then $\mathbf{y}_r$ could be rewritten as

$$\mathbf{y}_r^T = \boldsymbol{\Lambda}\mathbf{u}^i + \mathbf{w}^T \quad (12)$$

Then, this received signal $\mathbf{y}_r$ is multiplied by $\mathbf{b}^{(z)} = e^{-j2\pi v(z)[0:M-1]/M}$ for $z = (0,1,2 \ldots\ldots, Z)$ and for each sequence, the training signals are extracted, and the error rate between detected and transmitted training signals are calculated and averaged. Then the optimum index z_{opt} that minimizes the error is chosen. And $v(z_{opt})$ will represent the estimated normalized frequency offset. Let the pilot value $=A$, and define $\boldsymbol{\Upsilon} = onez(Z, 1)$. The mask matrix which filters out the data symbols and keep the pilot symbols could be written as:

$$\Psi_{p,q} = \begin{Bmatrix} 1 & if\ p,q \in \wp \\ 0 & otherwise \end{Bmatrix} \quad (13)$$

Let $\{a_z(k)\}_{k\in M,\, z\in Z}$ is the zth row of matrix $\boldsymbol{\Gamma}$. $\forall\ a_z \in \mathrm{C}^{\boldsymbol{n}}$ and $\boldsymbol{\Lambda} \in \mathrm{C}^{\boldsymbol{n}\times\boldsymbol{n}}$ $\exists$ for one of the Z sequences

$$\lim_{N\to\infty} a_z\{\boldsymbol{\Psi}\mathbf{y}_{eq}^T\} = \boldsymbol{Ones}\{\boldsymbol{\Psi}\mathbf{u}^T\}$$

because $\boldsymbol{\Psi}\mathbf{u}^T$ =Pilots values and positions then

$$\lim_{N\to\infty} a_z\{\boldsymbol{\Psi}\mathbf{y}_{eq}^T\} = AP$$

However, M is limited, not infinity, therefore error is occurred

$$a_z\{\boldsymbol{\Psi}\mathbf{y}_{eq}^T\} = \mathrm{Ones}\{\boldsymbol{\Psi}\mathbf{u}^T\} + \mathrm{err}$$

And this error depends on the difference between actual ε and the estimated CFO $\tilde{\varepsilon}$ which is equal to the closest tap in the generated subspace sequence

$$err = \left\{e^{-\frac{j2\pi(\varepsilon-\tilde{\varepsilon})[0:M-1]}{M}}(\boldsymbol{\Psi}\mathbf{u}^T) - \{\boldsymbol{\Psi}\mathbf{u}^T\}\right\}/P$$

Where $\varepsilon - \tilde{\varepsilon} \leq \frac{\text{subspace}}{2}$

$$\mathbf{Err}^T = \frac{(\boldsymbol{\Gamma\Psi}\mathbf{y}^T - PA\boldsymbol{\Upsilon})}{P}$$

$$v(z_{opt}) = \min_{v(z_{opt})}[\arg(err(z_{opt}))]$$

Then the detected signal could be expressed as

$$\mathbf{y}_{det} = \mathbf{y}.\mathbf{b}^{(z_{opt})} \quad (14)$$

It is clearly now, the effect of frequency offset on received signal depends on the difference between the original frequency offset and estimated frequency offset $(\varepsilon - z_{opt})$. Then the data signal of length N is extracted from (22) to get $\tilde{\mathbf{x}}$ and X transform is applied on $\tilde{\mathbf{x}}$ to collect the complex data signal $\tilde{\mathbf{d}}$.

$$\tilde{\mathbf{d}}^T = \mathbf{F}\tilde{\mathbf{x}}^T \quad (15)$$

B. *Frequency domain scenario*

In this scenario, the pilots are added in frequency domain before the IFFT. The frequency domain synchronization algorithm is shown in Fig. 3. The received signal is first multiplied by az, processed by FFT and then pilot samples are extracted and compared with the transmitted one. the previous procedures are done Z times and the frequency offset index that corresponding to the minimum error is taken and used in synchronization. It is clear that the processing time delay in this case is much more than the delay in the case of time domain CFO estimation. That because the need of use FFT Z times in the case of frequency domain synchronization.

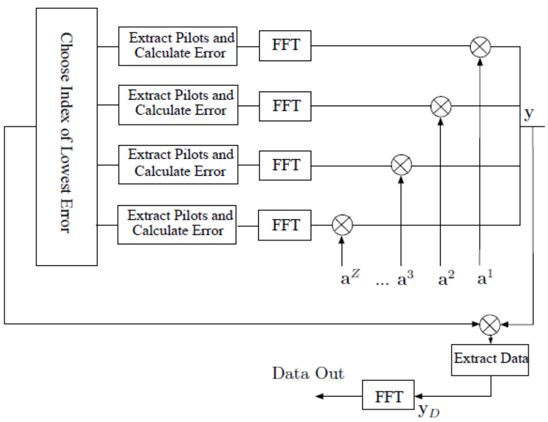

Fig. 3. System block diagram in the presence of CFO cancellation algorithm

IV. SPECTRAL EFFICIENCY

Both pilot symbols used in frequency domain CFO estimation and compensation and training symbols used in time domain CFO estimation and compensation are known information and lead to decreasing the spectral efficiency. Let us defined the spectral efficiency as the number of useful transmitted symbols N_d divided by the total transmitted symbols N_t, then the efficiency could be written as $\zeta = N_d/N_t$. In the case of frequency domain CFO, N_d=960, and N_t=1024, so, $\zeta = 93.75\%$. While, in the case of time domain CFO estimation N_d=1024, and N_t=1093, then $\zeta = 93.69\%$.

V. SIMULATION RESULTS AND DISCUSSION

The BER performance of the OFDM system in the presence of CFO and proposed synchronization algorithm is evaluated in this section. In this simulation, number of subcarrier used to carry the data N is 1024, number of pilots P is 69 and each tail is designed as one pilot sample followed by 16 data samples. Consequently, the overall transmitted OFDM symbol is 1093 samples in length. Additive white Gaussian noise (AWGN) channel is used in this simulation. Fig. 4 shows the of OFDM systems sensitivity to the CFO when no synchronization algorithm is applied. It is clear that the OFDM system completely loss the orthogonality when CFO normalized to subcarrier spacing is about 0.1. Fig. 5 shows the minimum mean-square error (MSE) between the actual and estimated carrier frequency offset in comparison with M&M [16] method when 16 QAM constellation is used and over the AWGN channel. It is evident the superiority of the proposed algorithm in this case. Fig. 6. shows the BER performance of OFDM system in the presence of carrier frequency offset " = 1.826 and the employment of the proposed synchronization algorithm over AWGN channel. It is worth mention here the proposed algorithm is also applicable in the case of multipath channel under assumption of perfect channel state information (CSI).

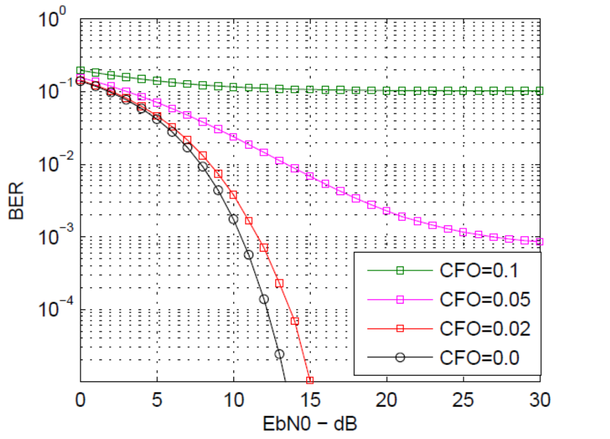

Fig. 4. BER performance of the OFDM system over the AWGN channel showing the system sensitivity to the CFO.

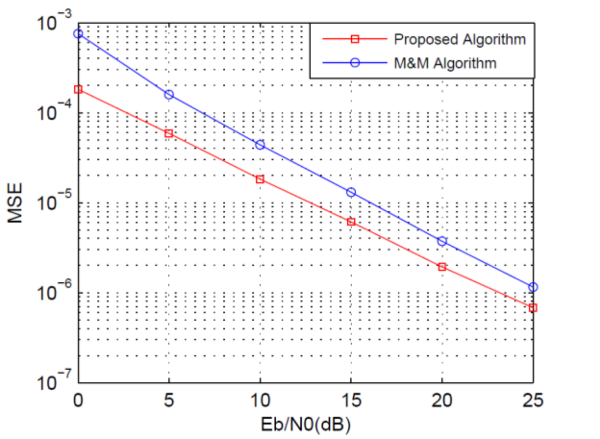

Fig. 5. Mean-square-error (MSE) of the proposed algorithm in comparison with the MSE of the $M\&M$ [16] algorithm.

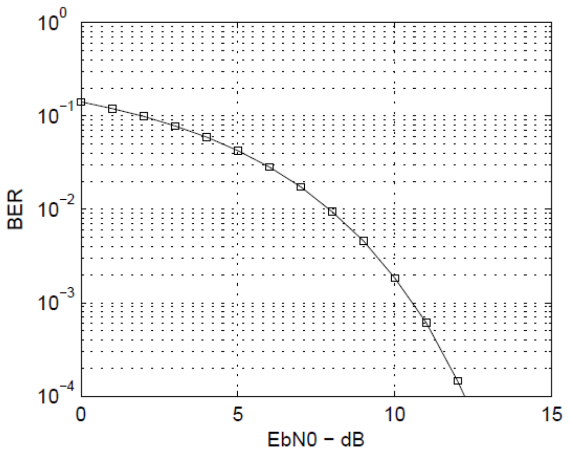

Fig. 6. BER performance of the OFDM system in the presence of the CFO and using the proposed Algorithm for CFO estimation and cancellation.

VI. CONCLUSION

In This paper, the challenging CFO synchronization problem was investigated and efficiently treated. The proposed CFO estimation technique was performed based on a known OFDM training sequence that is inserted between transmitted data samples where CFO was estimated sequentially. An iterative CFO estimation algorithm was proposed to calculate the minimum mean square error between transmitted training samples and received training samples and determines the corresponding CFO. The simulation results over AWGN channel showed the new technique effectiveness to estimate fractional and integer CFO. Furthermore, the elastic of the technique to be employed in time and frequency domain has been evinced. CFO was accurately estimated and orthogonality aggressively maintained at the receiver side.

VII. REFERENCES

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