

# FINITE ELEMENT ANALYSIS OF AUTOFRETTAGE PROCESS OF THICK – WALLED CYLINDERS

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## ABSTRACT

*The process of producing residual stresses in a thick-walled cylinder before it is put into usage is called Autofrettage, which it means; a suitable large enough pressure to cause yielding within the wall, is applied to the inner surface of the cylinder and then removed. So that a compressive residual stresses are generated to a certain radial depth at the cylinder wall.*

*The objective of the present study, is to investigate the effect of autofrettage process on radial, circumferential and total stresses by using von Mises yield criteria. Numerical simulation carried out on ABAQUS software to investigate the stresses distribution and calculate the autofrettage radius. The results reveal that, the autofrettage process of thick-wall cylinder lead to decrease the hoop and maximum von Mises stresses and relocate them from the inner surface of the cylinder to somewhere along it's thickness. The reduction in maximum stresses is strongly depend on autofrettage pressure, it was varying from ( 3.6% at  $P_{\text{autofrettage}} = 105 \text{ MPa}$  to 19.2% at  $P_{\text{autofrettage}} = 130 \text{ MPa}$  ) Also, it has been found, there is no effect of number of autofrettage stages on both of maximum von Mises stress and autofrettage radius.*

**Key words:** autofrettage, radial, hoop and axial stresses, von Mises yield criteria, autofrettage radius, optimum autofrettage pressure.

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## 1. INTRODUCTION

The wide applications of pressurized cylinder in chemical, nuclear, armaments, fluid transmitting plants, power plants and military equipment, in addition to the increasing scarcity and high cost of materials lead

the designers to concentrate their attentions to the elastic – plastic approach which offers more efficient use of materials [1, 2]. The process of producing residual stresses in the wall of thick-walled cylinder before it is put into usage is called *Autofrettage*, which it means; a suitable large enough pressure to cause yielding within the wall, is applied to the inner surface of the cylinder and then removed. So that a compressive residual stresses are generated to a certain radial depth at the cylinder wall. Then, during the subsequent application of an operating pressure, the residual stresses will reduce the tensile stresses generated as a result of applying operating pressure [1,3].

The effect of residual stresses on load-carry capacity of thick-walled cylinders have been investigate by Amran Ayob and Kabashi Albasheer [4], using both analytical and numerical techniques. The results of the study reveal three scenarios in the design of thick-walled cylinders. Amran Ayob and M. Kabashi Elbasheer [5], used von Mises and Tresca yield criteria to develop a procedure in which the autofrettage pressure determined analytically resulting in a reduced stress concentration. Then they compared the analytical results with FEM results. They concluded that, the autofrettage process increase the maximum allowable internal pressure but it cannot increase the maximum internal pressure to case whole thickness of the cylinder to yield. Noraziah et al. [6] presented an analytical autofrettage procedure to predict the required autofrettage pressure of different levels of allowable pressure and they validate their results with FEM results. They found three cases of autofrettage in design of pressurized thick – walled cylinders.

Ruilin Zhu and Jinlai Yang [7], by using both yield criteria von Mises and Tresca, presented an analytical equation for optimum radius of elastic-plastic junction in autofrettage cylinder, also they studied the influence of autofrettage on stress distribution and load bearing capacity. They concluded, to achieve optimum radius of elastic – plastic junction, an autofrettage pressure a bit larger than operating pressure should be applied before a pressure vessel is put into use. Zhong Hu and Sudhir Puttagunta [8] investigate the residual stresses in the thick- walled cylinder induced by internal autofrettage pressure, also they found the optimum autofrettage pressure and the maximum reduction percentage of the von Mises stress under elastic-limit working pressure. Md. Tanjin Amin et al. [9] determined the optimum elasto – plastic radius and optimum autofrettage pressure by using von Mises yield criterion , then they have been compared with Zhu and Yang's model [8]. Also they observed that the percentage of maximum von Mises stress reduction increases as value of radius ratio (K) and working pressure increases. F. Trieb et al. [10] discussed practical application of autofrettage on components for waterjet cutting. They reported that the life time of high pressure components is improved by increasing autofrettage depth due to reduction of tangential stress at inner diameter, on other hand too high pressure on outside diameter should be avoided to prevent cracks generate. In addition to determine the optimum autofrettage pressure and the optimum radius of elastic-plastic junction , Abu Rayhan Md. et al.[11] evaluated the effect of autofrettage process in strain hardened thick – walled pressure vessels by using equivalent von Mises stress as yield criterion. They found, the number of autofrettage stages has no effect on maximum von Mises stress and pressure capacity. Also, they concluded that, optimum autofrettage pressure depends on the working pressure and on the ratio of outer to inner radius.

## 2. PRESSURE LIMITS AND STRESS DISTRIBUTION IN NON – AUTOFRETTAGED CYLINDER

### 2.1. Pressure Limits of Non – Autofrettage Cylinder

According to von Mises yield criterion, Both of the internal pressure requires to yield the inner surface of the cylinder ( i.e. partial autofrettage ),  $P_{Yi}$  , and that to yield the whole wall of the cylinder ( i.e. completely autofrettage ),  $P_{Yo}$  , can be calculated from equations ( 1 & 2 ) [4, 7]

$$P_{Yi} = \frac{\sigma_y (K^2 - 1)}{\sqrt{3}K^2} \quad (1)$$

$$P_{Yo} = \frac{\sigma_y (K^2 - 1)}{\sqrt{3}} \quad (2)$$

## 2.2. Stress Distribution of Non – Autofrettage Cylinder

The radial stress  $\sigma_r$ , circumferential ( hoop ) stress  $\sigma_\theta$  and axial stress  $\sigma_z$ , distributions in non – autofrettage cylinder subjected to an operating pressure,  $P_i$ , are given by Lamé's formulations which is available in [3, 4, 5, 6, 7 ]. As shown in Figure ( 1 ), it is clear that the tensile hoop,  $\sigma_\theta$ , compressive radial,  $\sigma_r$ , and maximum von Mises stresses have their maximum values at the inner surface of the cylinder. The hoop stress has always positive value which represents as tensile stress while the stress in the radial direction is always compressive. Also the hoop tensile stress's value is greater than radial compressive stress's value.

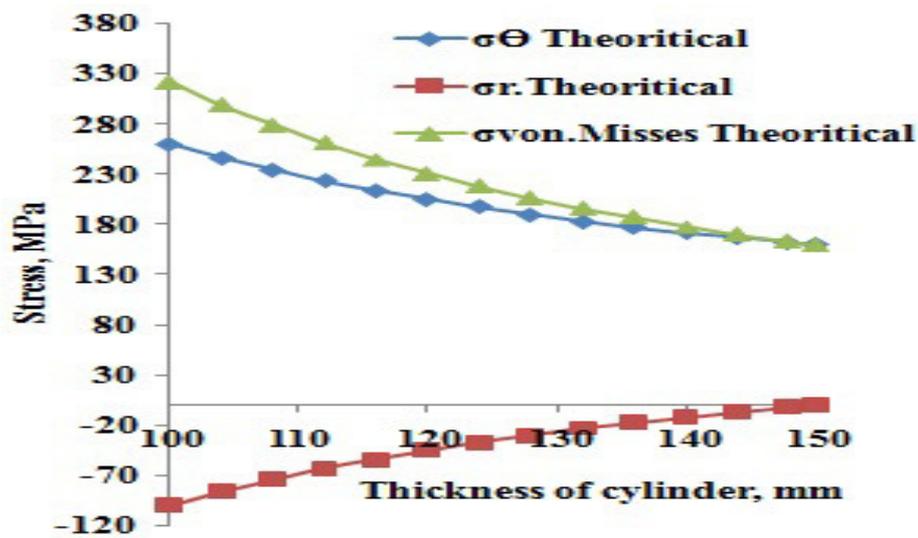


Figure 1 Stress Distribution on Non-Autofrettage thick-walled Cylinder Subjected to Operating Pressure

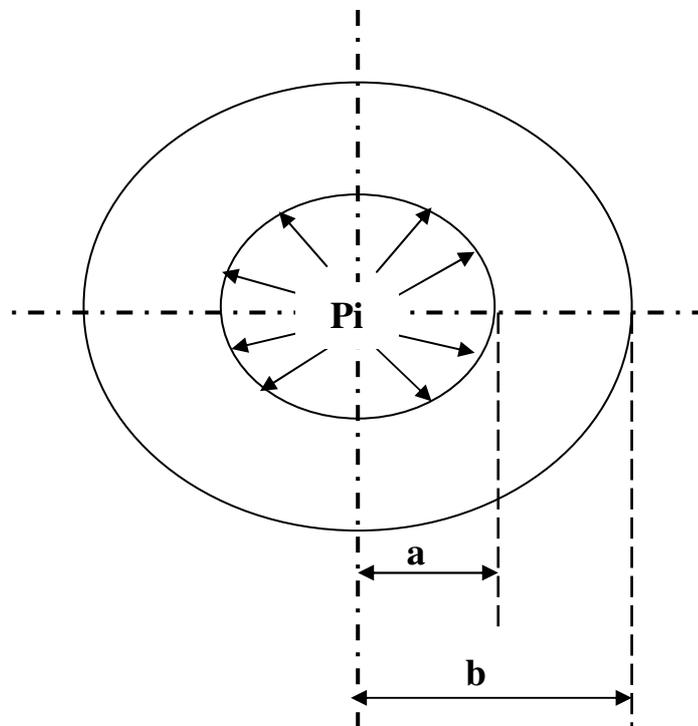


Figure 2 Geometry of Investigated Model

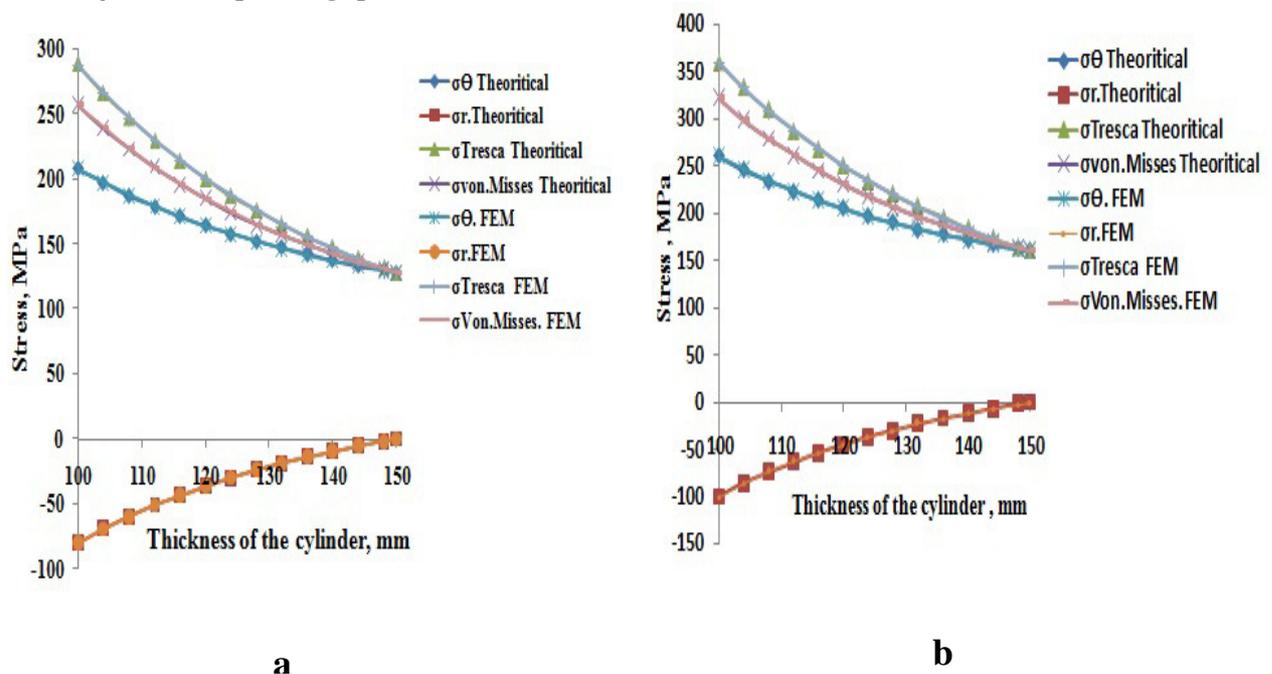
### 3. FINITE ELEMENT ANALYSIS AND MATERIALS OF NUMERICAL SIMULATION MODELS

Figure ( 2 ) illustrates the geometry of investigated cylinder that is made up of carbon steel with young's modulus of ( 203 GPa ), Poisson's ratio of ( 0.33 ) and yield stress of ( 325 MPa ) [ 12 ]. It subjected to internal pressure (  $P_i$  ). The material is assumed homogeneous and isotropic. To compute the required results, Numerical simulation is carried out on ABAQUS ver.6.9 [13]. The investigated cases are consider as 2D – planar problem and quadratic element have been used ( CPS8R–8– nodes )

### 4. VALIDATION OF NUMERICAL SIMULATION

In the present study, the validation of software has been done by comparing the analytical calculation results which obtained by solutions of equations are available in literatures [ 3, 4, 5, 6 7], with results of numerical solution using ABAQUS ver.6.9.

From figure ( 3 ) , it is clear that, the theoretical and numerical calculations of circumferential, radial and maximum von Mises stresses for different internal pressure are very closed and overlap each other. It means, a good agreement is found between the results, and the static analysis shows that, the percentage of errors between the result of analytical and numerical solution are les than 0.5%. This low percentage of errors affirms, there are no significsnt differences between the theoretical results and those obtained by simulation. Consequently, FE modeling using ABAQUS software can be used to study the effect of autofrettage process on the stress distribution and location of autofrettage radius (  $R_a$  ) of thick–walled cylinder subjected to operating pressure.



**Figure 3** Validation of Numerical Solution Results with Theoretical results at different operating pressure; **a** - Operating pressure = 80 MPa, **b** – Operating pressure = 100 MPa.

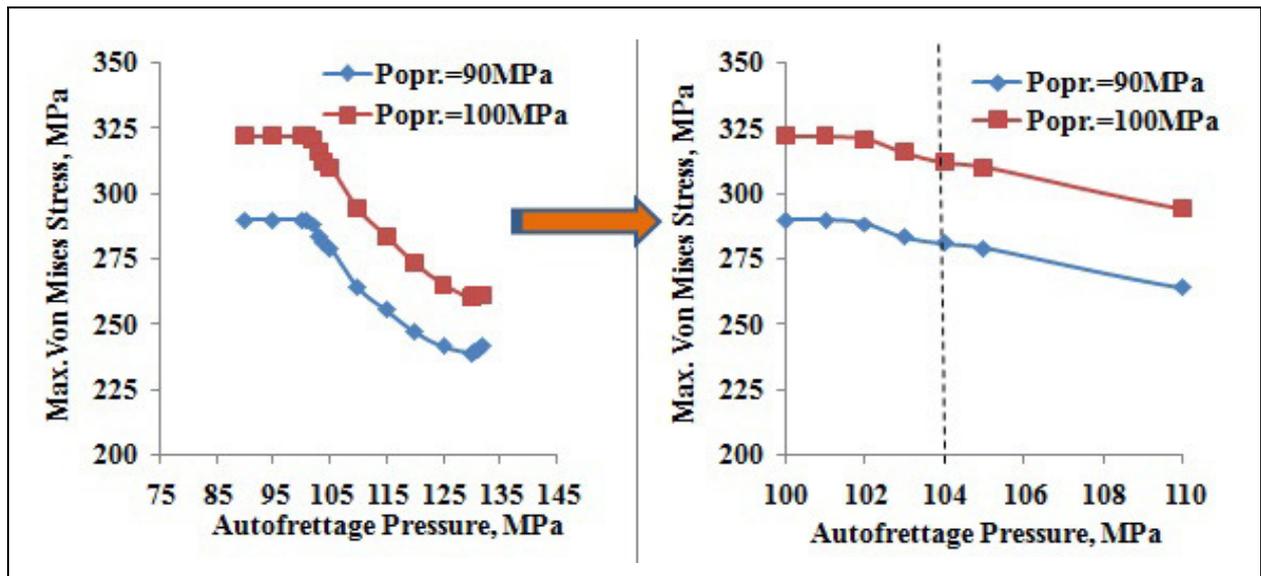
## 5. RESULTS AND DISCUSSIONS

### 5.1. Minimum Autofrettage Pressure

By calculating the minimum pressure that needed to yield the inner surface of the tested cylinder (  $P_{Y_i}$  ) from equation (1) , it was found equal to ( 104.243 Mpa ). That is mean, the effect of autofrettage pressure will start at (104.243 MPa), then the plastic deformation spreads through the cylinder thickness. Figure (4) shows that, the simulation solution of effect of autofrettage pressure on maximum von Mises stress for

different operating pressure, it is clear that , there is no effect of autofrettage pressure on maximum von Mises stress generating in the cylinder due to the operating pressure as long as it is less than ( 104 MPa ) for both value of operating pressure. Then , when it is exceed (  $P_{\text{autofrettage}} > 104 \text{ MPa}$  ) the maximum Von Mises stress decreases depending on the autofrettage pressure, the bigger value of autofrettage pressure, the lower of maximum von Mises stress.

In addition to that , it has been observed from Table 1 that, the maximum von Mises stress decreases with increasing the autofrettage pressure even  $P_{\text{autofrettage}}$  reache value of about ( 130 MPa ) then starts increasing, which it means, this value of autofrettage pressure represents the optimum autofrettage pressure [5,6]. This results agree with result was found by [ 1, 9, 11].



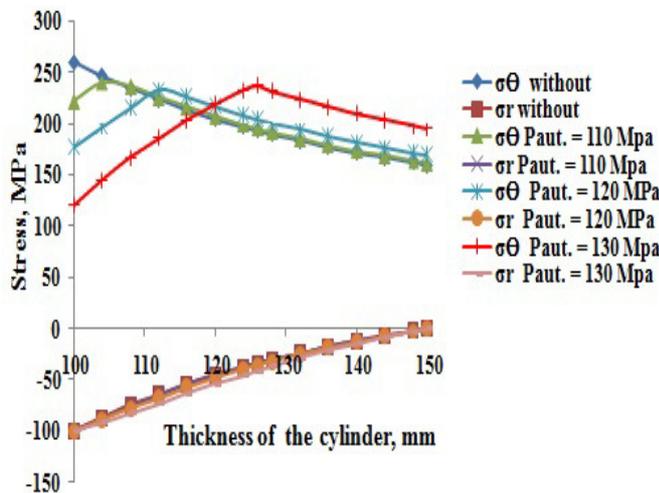
**Figure 4** Simulation solution results of autofrettage pressures' effect on Maximum von Mises stress at different operating pressure.

**Table 1** FEM results of effect of Autofrettage Pressure on Maximum von Mises Stress

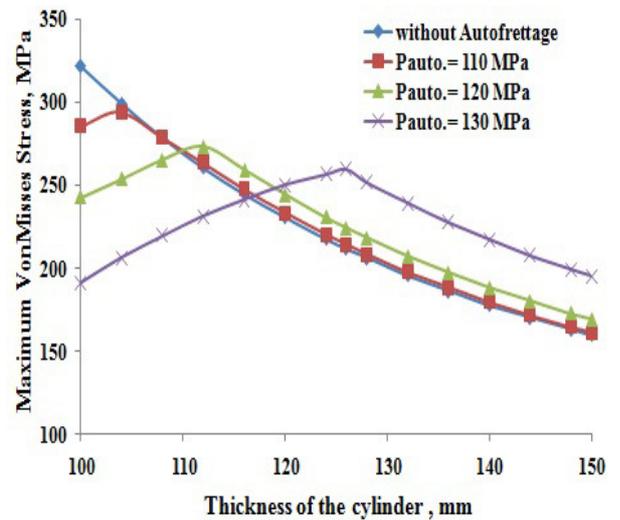
No.	Operating Pressure, MPa	Autofrettage Pressure, MPa	Max. vonMises Stress, MPa
1.	90	120	247.00
2.	90	125	241.40
3.	90	130	238.8
4.	90	131	240.20
5.	90	132	241.40
6.	100	120	273.10
7.	100	125	265.20
8.	100	130	260.00
9.	100	131	260.80
10.	100	132	261.00

### 5.2. Effect of Autofrettage Process on Stress Distribution

Figures ( 5, 6 & 7 ) demonstrates the effect of autofrettage process on stress distribution of thicked–walled cylinder subjected to operating pressure of ( 100 MPa ). It is obvious, the autofrettage process leads to decrease the value of maximum von Mises stress and relocated the compressive circumferential & maximum von Mises stresses from the inner surface of the cylinder to somewhere through it's thickness. This new location of maximum von Mises stress called *Autofrettage radius*,  $R_a$  . It does not depend on operating pressure while it is strongly affected by autofrettage pressure as shown in Table 2, which shows the values of autofrettage radius,  $R_a$  , with different values of autofrettage pressure. Also, it is found , the reduction in maximum von Mises stresses varying from ( 3.6 % at  $P_{\text{autofrettage}} = 105 \text{ MPa}$  ) to ( 19.2% at  $P_{\text{autofrettage}} = 130 \text{ MPa}$  ). It is vital to see that , there is no significant effect of autofrettage process on radial stress as that seen on the circumferential stress.



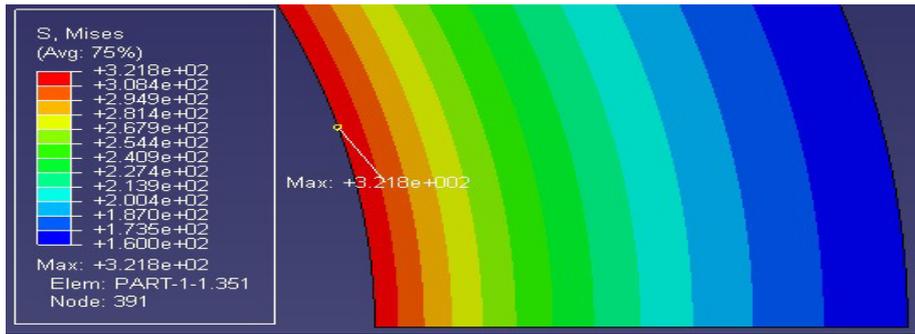
**Figure 5** Effect of Autofrettage Pr. on hoob & Radial stresses at operating Pressure = 100 MPa.



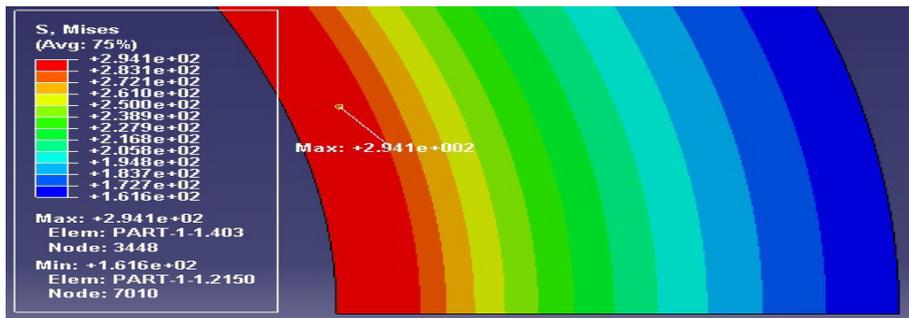
**Figure 6** Effect of Autofrettage Pr. on maximum von Mises stress at operating Pressure = 100 MPa.

**Table 2** FEM results of effect of Autofrettage Pressure on Maximum Von Mises Stress

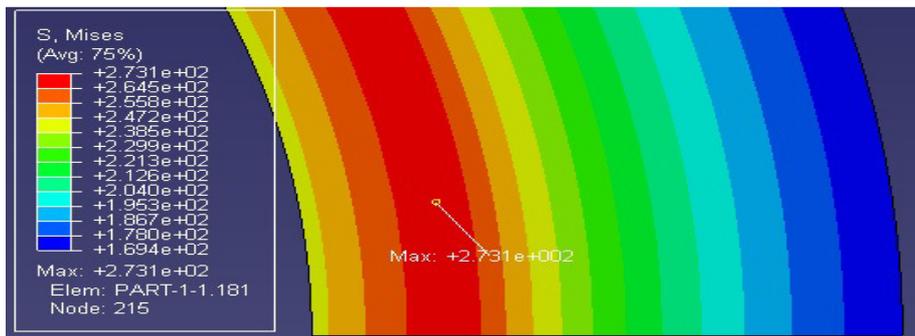
No.	Operating Pressure, MPa	Autofrettage Pressure, MPa	Max. von Mises Stress, MPa	Autofrettage Radius, mm	Reduction in Maximum von Mises stress %
1.	90	without	290.00	100	--
2.	90	105	278.975	101.99836	3.8 %
3.	90	110	264.108	103.99686	8.9 %
4.	90	120	246.88	111.9915	14.8 %
5.	90	130	238.792	125.9761	17.65 %
6.	100	without	321.836	100	--
7.	100	105	310.00	101.99836	3.6 %
8.	100	110	294.020	103.99686	8.6 %
9.	100	120	273.116	111.9915	15.2 %
10.	100	130	259.992	125.9761	19.2 %



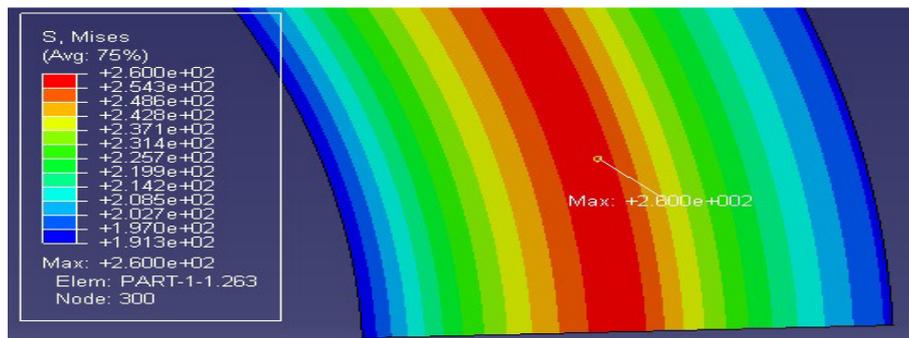
A



B



C



D

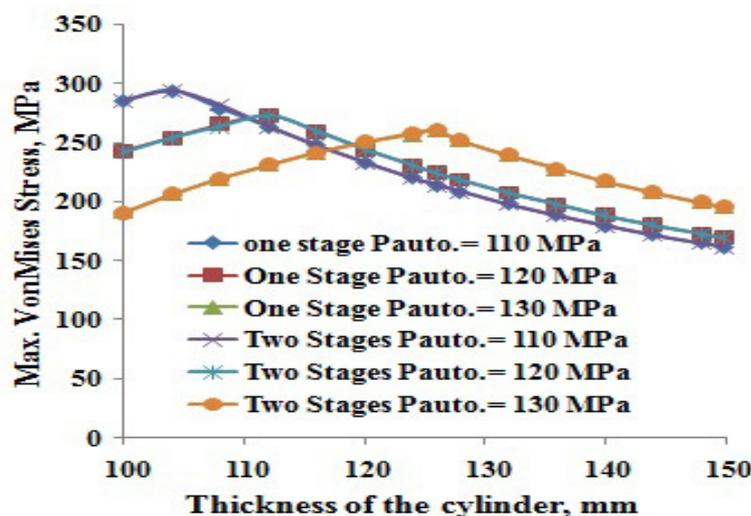
**Figure 7** FE analysis of effect of autofrettage Pressure on maximum von Mises stress and location of autofrettage radius at operating Pressure = 100 MPa ; **A** - without autofrettage, **B** -  $P_{\text{autofrettage}} = 110$  MPa, **C** -  $P_{\text{autofrettage}} = 120$  MPa, **D** -  $P_{\text{autofrettage}} = 130$  MPa.

### 5.3. Effect of Autofrettage Stages on Maximum Von Mises Stress

To investigate the effect of autofrettage stages on maximum von Mises stress, the investigated cylinder was subjected to ( 100 MPa ) as operating pressure and autofrettage pressures of ( 110, 120 and 130 MPa ) are done by two steps, at first step, the autofrettage pressure has been applied in one stage, while at second step it was done by three loading stages ( see Table 3 ). As can be noticed clearly in Table 3 and Figure ( 7 ), the numerical results confirm there is no effect of autofrettage stages on the maximum Von Mises stress generated in the cylinder due to operating pressure. This results are very close to the with results have been found by [3].

**Table 3** FEM results of effect of Autofrettage stages on Maximum Von Mises Stress

No. of case	Autofrettage pressure, MPa First stage	Unloading step MPa	Autofrettage pressure, MPa second stage	Unloading step MPa	Loading of Operating Pressure, MPa	Maximum von Mises Stress, MPa
Case I	110	0	-	-	100	294.020
Case II	120	0	-	-	100	273.116
Case III	130	0	-	-	100	259.992
Case IV	105	0	110	0	100	294.033
Case V	105	0	120	0	100	273.05
Case VI	105	0	130	0	100	260.254



**Figure 7** Numerical solution results of effect of autofrettage stage on Max. von Mises stresses and autofrettage radius at operating Pressure = 100 Mpa

## 6. CONCLUSION

The results of present investigation can be summarized as

- The autofrettage process on thick-walled cylinder leads to decrease the circumferential and maximum von mises stresses and relocate them from the inner surface of the cylinder to somewhere along it's thickness, which called as, autofrettage radius,  $R_a$ .
- The autofrettage radius,  $R_a$ , is strongly affected by autofrettage pressure while it does not depend on the operating pressure..

- There is no effect of autofrettage stages on maximum Von Mises stress developed in the cylinder subjected to an operating pressure.

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